

# symmetry and Liouville<sup>†</sup> equation superextensions<sup>\*</sup>

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## Abstract

It is shown that the method of nonlinear realization of local supersymmetry being applied to the  $n = (1, 1)$  superconformal symmetry allows one reduce the new version of the super-Liouville equation to the ordinary one owing to the relaxation of the auxiliary equation of motion fixing the gauge parameters.

## 1 Introduction

It was reviled in [1] that the general solution of the string- inspired nonlinear equations describing the intrinsic geometry of the bosonic string worldsheet in the geometrical approach [2, 3, 4] can be represented in terms of the two sets of left- and right-moving Lorentz harmonic variables. The latter are defined as the coordinates of compact coset space isomorphic to the  $D$ -dimensional sphere

$$S_{D-2} = \frac{SO(1, D-1)}{SO(1, 1) \times SO(D-2) \times K_{D-2}}, \quad (1)$$

with the Borel subgroup of Lorentz group in the denominator of the fraction (1). A natural way of extending these results on the case of superstring is to search for the corresponding solutions of supersymmetric equations of motion describing the embedding of superstrings into the flat target superspaces. The simplest example of such a pattern of equations inherent to the  $N = 2$ ,  $D = 3$  superstring is the new version of the  $n = (1, 1)$  super-Liouville equation [5]<sup>1</sup>

$$D_- D_+ W = e^{2W} \Psi_L^+ \Psi_R^-, \quad D_\pm = \partial_\pm + i\eta^\pm \partial_{\pm\pm}, \quad (2)$$

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<sup>1</sup>The old version given in [6, 7] is also acceptable.

$$\begin{aligned} D_+ \Psi_L^+ - 2(D_+ W) \Psi_L^+ &= 1, \\ D_- \Psi_R^- - 2(D_- W) \Psi_R^- &= 1, \end{aligned} \quad (3)$$

where

$$W(\xi^{\pm\pm}, \eta^\pm) = u(\xi^{\pm\pm}) + i\eta^+ \psi^-(\xi^{\pm\pm}) + i\eta^- \psi^+(\xi^{\pm\pm}) + i\eta^- \eta^+ F(\xi^{\pm\pm}), \quad (4)$$

$$\begin{aligned} \Psi_L^+(\xi^{++}, \eta^+) &= \omega^+(\xi^{++}) + \eta^+ F_L(\xi^{++}), \\ \Psi_R^-(\xi^{--}, \eta^-) &= \omega^-(\xi^{--}) + \eta^- F_R(\xi^{--}), \end{aligned} \quad (5)$$

are the worldsheet superfields.

In this report we would like to show that as well as in the bosonic case Eqs. (2), (3) can be solved exactly in terms of the Lorentz harmonics variables parametrizing the coset space (1) but unlike to them valued on the worldsheet superspace  $\mathbf{R}^{(2|2)} = \{\xi^{++}, \eta^+; \xi^{--}, \eta^-\}$  and restricted by the special covariant constraints deriving from the nonlinear realization of the superconformal symmetry.

## 2 Gauge and superconformal symmetries

### 2.1 Linear realization

It is not hard to verify that the flat spinor covariant derivatives entering the Eqs. (2), (3) are transformed homogeneously

$$D'_\pm = (D_\pm \eta^{\pm'})^{-1} D_\pm. \quad (6)$$

with respect to the two copies of one dimensional superconformal transformations

$$\begin{aligned} \xi^{\pm\pm'} &= \xi^{\pm\pm} + a^{\pm\pm} + i\eta^\pm \epsilon^\pm \sqrt{1 + a^{\pm\pm'}}, & a^{\pm\pm'} &= \partial_{\pm\pm} a^{\pm\pm} \\ \eta^{\pm'} &= \epsilon^\pm + \eta^\pm \sqrt{1 + a^{\pm\pm'} + i\epsilon^\pm \epsilon^{\pm'}}, & \epsilon^{\pm'} &= \partial_{\pm\pm} \epsilon^\pm, \end{aligned} \quad (7)$$

restricted by the condition

$$D_\pm \xi^{\pm\pm'} - i\eta^\pm D_\pm \eta^{\pm'} = 0. \quad (8)$$

This indicate that the following gauge transformations of superfields

$$\begin{aligned} W'(\xi^{\pm\pm'}, \eta^{\pm'}) &= W(\xi^{\pm\pm}, \eta^\pm) - \frac{1}{4} \ln(D_+ \eta^{+'}) - \frac{1}{4} \ln(D_- \eta^{-'}), \\ \Psi_L^{+'}(\xi^{++'}, \eta^{+'}) &= (D_+ \eta^{+'})^{-1/2} \Psi_L^+(\xi^{++}, \eta^+), \\ \Psi_R^{-'}(\xi^{--'}, \eta^{-'}) &= (D_- \eta^{-'})^{-1/2} \Psi_R^-(\xi^{--}, \eta^-), \end{aligned} \quad (9)$$

leaves intact the form of the Eq. (2). At the same time the second Eq. (3) is not changed as well only when gauge is completely fixed

$$(D_- \eta'^{-})^{-3/2} = 1. \quad (10)$$

It was shown in [5] that this gauge condition impose very essential restrictions on the superfields (4), (5) removing all their components excepting leading once  $u(\xi^{\pm\pm})$  and  $\omega^\pm(\xi^{\pm\pm})$ . The simplest way of achieving this result is to transit to the nonlinear realization of superconformal symmetry in which the gauge fixing Eq. (3) can be initially something relaxed and then exactly solved. In the next Section we are going to construct this realization following closely to Ref. [9].

## 2.2 Nonlinear realization

Let us suppose that the v.e.v. of the component fields  $F_L(\xi^{++})$  and  $F_R(\xi^{--})$  in (5) are not equal to zero and as consequence of this the local supersymmetry (7) is actually spontaneously broken. In this case the fermionic components  $\omega^\pm(\xi^{\pm\pm})$  acquire the meaning of the corresponding Goldstone fermions and one can exploit them for a singling out of the ordinary Liouville equation from the system (2), (3) in a manifestly covariant manner. Indeed, it is well-known that in the models with spontaneously broken supersymmetry all the SFs becomes reducible [8], [9]. Their irreducible parts are transformed, however, universally with respect to the action of the original supergroups, as the linear representations of the underlying unbroken subgroups but with the parameters depending nonlinearly on the Goldstone fermions. There is always the possibility to impose on these SFs some absolutely covariant restrictions providing to remove out from them undesirable degrees of freedom. Here we can take advantage of possibilities of this approach for deriving the relevant solution of the Eqs. (2), (3).

For the beginning let us consider some special aspects of the nonlinear realization of superconformal symmetry in superspace. As was shown in Refs. [10], [9] for this purpose we need firstly splits the general finite element of the group (7)

$$G(\zeta) \equiv \zeta', \quad (11)$$

where  $\zeta = \{\xi^{\pm\pm}, \eta^\pm\}$ , onto the product of elements of two successive transformations

$$G(\zeta) = K(G_0(\zeta)). \quad (12)$$

In Eq. (12) the following standard notations are used. As before the  $G_0(\zeta)$  denotes the "primes" coordinates  $\zeta'$  but index *zero* means that they referring now only to the stability subgroup

$$\begin{aligned}\xi^{\pm\pm'} &= \xi^{\pm\pm} + a^{\pm\pm}(\xi^{\pm\pm}), \\ \eta^{\pm'} &= \eta^{\pm} \sqrt{1 + \partial_{\pm\pm} a^{\pm\pm}}.\end{aligned}\tag{13}$$

By the definition we suppose also that the stability subgroup includes only the ordinary conformal transformations (parameters  $a^{\pm\pm}(\xi^{\pm\pm})$ ) of the bosonic coordinates  $\xi^{\pm\pm}$  and the special scale transformations (parameters  $\sqrt{1 + \partial_{\pm\pm} a^{\pm\pm}}$ ) of the fermionic coordinates  $\eta^{\pm}$ . Note, that the first multiplier in the decomposition (12) is easily recognized as the representatives of the left coset space  $G/G_0$

$$\begin{aligned}K^{\pm\pm}(\zeta) &= \xi^{\pm\pm} + i\eta^{\pm}\epsilon^{\pm}(\xi^{\pm\pm}), \\ K^{\pm}(\zeta) &= \epsilon^{\pm}(\xi^{\pm\pm}) + \eta^{\pm} \sqrt{1 + i\epsilon^{\pm}\partial_{\pm\pm}\epsilon^{\pm}}.\end{aligned}\tag{14}$$

It deserves to mention that in the decomposition (12) the comultipliers  $K$  and  $G_0$  are chosen in such a way that the irreducibility constraint (8) is satisfied separately for each of them.

The prescription of constructing the corresponding nonlinear realization proposed in [9] is as follows. Let us identify the local parameters  $\epsilon^{\pm}(\xi^{\pm\pm})$  in (14) with the Goldstone fields  $\lambda^{\pm}(\xi^{\pm\pm})$

$$\begin{aligned}\tilde{K}^{\pm\pm}(\tilde{\zeta}) &= \tilde{\xi}^{\pm\pm} + i\tilde{\eta}^{\pm}\lambda^{\pm}(\tilde{\xi}^{\pm\pm}), \\ \tilde{K}^{\pm}(\tilde{\zeta}) &= \lambda^{\pm}(\tilde{\xi}^{\pm\pm}) + \tilde{\eta}^{\pm} \sqrt{1 + i\lambda^{\pm}\tilde{\partial}_{\pm\pm}\lambda^{\pm}}.\end{aligned}\tag{15}$$

and take for  $\tilde{K}(\tilde{\zeta})$  the transformation law associated to (12)

$$G(\tilde{K}(\tilde{\zeta})) = \tilde{K}'(\tilde{G}_0(\tilde{\zeta})).\tag{16}$$

In Eq. (16) the newly introduced coordinates  $\tilde{\zeta} = \{\tilde{\xi}^{\pm\pm}, \tilde{\eta}^{\pm}\}$  are transformed differently as compared with  $\zeta = \{\xi^{\pm\pm}, \eta^{\pm}\}$  in (11). Indeed, in accordance with (13) they change only under the vacuum stability subgroup

$$\begin{aligned}\tilde{\xi}^{\pm\pm'} &= \tilde{\xi}^{\pm\pm} + \tilde{a}^{\pm\pm}(\tilde{\xi}^{\pm\pm}), \\ \tilde{\eta}^{\pm'} &= \tilde{\eta}^{\pm} \sqrt{1 + \tilde{\partial}_{\pm\pm} \tilde{a}^{\pm\pm}}.\end{aligned}\tag{17}$$

where the parameters  $\tilde{a}^{\pm\pm}(\tilde{\xi}^{\pm\pm})$  turn out to be dependent nonlinearly on the Goldstone fields  $\lambda^{\pm}(\xi^{\pm\pm})$  and its derivatives. Eqs. (16) and (17) determine the transformation properties of the Goldstone fermions  $\lambda^{\pm}(\xi^{\pm\pm})$  with respect to the nonlinear realization of the superconformal group  $G$  in coset space (15).

### 3 Splitting superfields and gauge relaxing

Up to now we have dealt with only formal prescription of construction of the nonlinear realization of superconformal group  $G$  without any relation of this procedure to the original equations (2), (3). Nevertheless, there is the simple possibility to gain a more deeper insight into the model we started with if we compare two Eqs. (11) and (16). We find that  $\tilde{K}(\tilde{\zeta})$  transform under  $G$  in precisely the same manner as the initial coordinates  $\zeta$  of superspace  $\mathbf{R}^{(2|2)}$ . Thus we have the unique possibility to identify them

$$\zeta = \tilde{K}(\tilde{\zeta}). \quad (18)$$

Eq. (18) establishes the relationship between two forms of the realization of superconformal symmetries in superspace, i.e. linear and nonlinear one. One of the remarkable futures of the transformations (18) is that superspace of the nonlinear realization  $\tilde{\mathbf{R}}^{(2|2)} = \{\tilde{\zeta}\}$  turns out to be completely "splitting" in virtue of the transformations (17) which are not mixed the bosonic and fermionic variables. Due to this very important fact the SFs of the nonlinear realization valued in  $\tilde{\mathbf{R}}^{(2|2)}$  becomes *reducible*. Furthermore we receive the unique possibility of relaxing the gauge fixing Eqs. (3) because it appears that in frame of the nonlinear realization there exist the new covariant objects consisting only on the Goldstone fields which transformed under the superconformal symmetry as the combinations of SFs standing in the l.h.s. of these equations. Indeed, let us consider the following quantities

$$\begin{aligned} \Phi_L^+(\xi^{++}, \eta^+) &\equiv \tilde{F}_L(\tilde{\xi}^{++})(\tilde{D}_+\eta^+)^{-3/2}, \\ \Phi_R^-(\xi^{--}, \eta^-) &\equiv \tilde{F}_R(\tilde{\xi}^{--})(\tilde{D}_-\eta^-)^{-3/2}. \end{aligned} \quad (19)$$

Immediately from the definitions and the connections between the spinor covariant derivatives of linear and nonlinear realizations  $D_\pm = (\tilde{D}_\pm \eta^\pm)^{-1} \tilde{D}_\pm$  one can check that these objects are transformed as a superconformal densities of the weight  $-3/2$  with respect to the superconformal transformations (7)

$$\begin{aligned} \Phi_L^{+'}(\xi^{++'}, \eta^{+'}) &= (D_+\eta^{+'})^{-3/2} \Phi_L^+(\xi^{++}, \eta^+), \\ \Phi_R^{-'}(\xi^{--'}, \eta^{-'}) &= (D_-\eta^{-'})^{-3/2} \Phi_R^-(\xi^{--}, \eta^-), \end{aligned} \quad (20)$$

if the fields of nonlinear realization  $\tilde{F}_L(\tilde{\xi}^{++})$ ,  $\tilde{F}_R(\tilde{\xi}^{--})$  are supposed to be transformed as the corresponding densities with the same weight with respect to ordinary conformal transformations

$$\tilde{F}_L'(\tilde{\xi}^{++'}) = \omega_L^{-3/2}(\tilde{\xi}^{++}) \tilde{F}_L(\tilde{\xi}^{++}), \quad \tilde{F}_R'(\tilde{\xi}^{--'}) = \omega_R^{-3/2}(\tilde{\xi}^{--}) \tilde{F}_R(\tilde{\xi}^{--}), \quad (21)$$

where

$$\omega_L \equiv \sqrt{1 + \partial_{++} a^{++}}, \quad \omega_R \equiv \sqrt{1 + \partial_{--} a^{--}}. \quad (22)$$

Therefore if we change the units in the r.h.s. of the Eqs. (3) on the SFs (19) we obtain the equations

$$\begin{aligned} D_+ \Psi_L^+ - 2(D_+ W) \Psi_L^+ &= \Phi_L^+(\xi^{++}, \eta^+), \\ D_- \Psi_R^- - 2(D_- W) \Psi_R^- &= \Phi_R^-(\xi^{--}, \eta^-), \end{aligned} \quad (23)$$

which will not restrict the gauge parameters. Now let us return to the Eq. (2). Performing the change of variables (18) in the Eqs. (2), (23) we get

$$\tilde{D}_- \tilde{D}_+ \tilde{W} = e^{2\tilde{W}} \tilde{\Psi}_L^+ \tilde{\Psi}_R^-, \quad \tilde{D}_\pm = \tilde{\partial}_\pm + i\tilde{\eta}^\pm \tilde{\partial}_{\pm\pm}, \quad (24)$$

$$\begin{aligned} \tilde{D}_+ \tilde{\Psi}_L^+ - 2(\tilde{D}_+ \tilde{W}) \tilde{\Psi}_L^+ &= \tilde{F}_L, \\ \tilde{D}_- \tilde{\Psi}_R^- - 2(\tilde{D}_- \tilde{W}) \tilde{\Psi}_R^- &= \tilde{F}_R, \end{aligned} \quad (25)$$

where the SFs and covariant derivatives of the nonlinear realization (16), (17) and (18) are introduced

$$\begin{aligned} W(\xi^{\pm\pm}, \eta^\pm) &= \tilde{W}(\tilde{\xi}^{\pm\pm}, \tilde{\eta}^\pm) - \frac{1}{4} \ln(\tilde{D}_+ \eta^+) - \frac{1}{4} \ln(\tilde{D}_- \eta^-), \\ \Psi_L^+(\xi^{++}, \eta^+) &= (\tilde{D}_+ \eta^+)^{-1/2} \tilde{\Psi}_L^+(\tilde{\xi}^{++}, \tilde{\eta}^+), \\ \Psi_R^-(\xi^{--}, \eta^-) &= (\tilde{D}_- \eta^-)^{-1/2} \tilde{\Psi}_R^-(\tilde{\xi}^{--}, \tilde{\eta}^-). \end{aligned} \quad (26)$$

Note that although the form of the Eq. (24) is precisely the same as the original one (2) the SFs of the nonlinear realization appearing in (24), (25) are distinguished drastically from the SFs of linear realization. As it follows from (17) the "new" SFs  $\tilde{W}$  and  $\tilde{\Psi}$  are transformed under the action of  $G$  only with respect to their stability subgroup (17)

$$\begin{aligned} \tilde{W}'(\tilde{\xi}^{\pm\pm'}, \tilde{\eta}^{\pm'}) &= \tilde{W}(\tilde{x}^{\pm\pm'}, \tilde{\eta}^{\pm'}) - \frac{1}{4} \ln(\tilde{D}_+ \tilde{\eta}^{+'}) - \frac{1}{4} \ln(\tilde{D}_- \tilde{\eta}^{-'}), \\ \tilde{\Psi}_L^{+'}(\tilde{\xi}^{++'}, \tilde{\eta}^{+'}) &= (\tilde{D}_+ \tilde{\eta}^{+'})^{-1/2} \tilde{\Psi}_L^+(\tilde{\xi}^{++}, \tilde{\eta}^+), \\ \tilde{\Psi}_R^{-'}(\tilde{\xi}^{--'}, \tilde{\eta}^{-'}) &= (\tilde{D}_- \tilde{\eta}^{-'})^{-1/2} \tilde{\Psi}_R^-(\tilde{\xi}^{--}, \tilde{\eta}^-). \end{aligned} \quad (27)$$

Substituting here the explicit form of gauge parameters deduced from the transformations (17)

$$\tilde{D}_\pm \tilde{\eta}^{\pm'} = \sqrt{1 + \tilde{\partial}_{\pm\pm} \tilde{a}^{\pm\pm}}, \quad (28)$$

one concludes that all the component fields of the SFs  $\tilde{W}$  and  $\tilde{\Psi}^\pm$  are transformed *independently* from each other. Thus we can put down the following manifestly covariant constraints

$$\tilde{W}(\tilde{\xi}^{\pm\pm}, \tilde{\eta}^\pm) = \tilde{u}(\tilde{\xi}^{\pm\pm}), \quad (29)$$

$$\tilde{\Psi}_L^+ = \tilde{\eta}^+ \tilde{F}_L \Rightarrow \tilde{\omega}^+ = 0, \quad \tilde{\Psi}_R^- = \tilde{\eta}^- \tilde{F}_R \Rightarrow \tilde{\omega}^- = 0. \quad (30)$$

It is instructive to note that the two last constraints in (30) are specific for the theories with spontaneously broken local symmetries. They establish the equivalence connections between the Goldstone fields of linear and nonlinear realizations. In the case under consideration one can prove that this connection between the corresponding fields  $\omega^\pm$  and  $\lambda^\pm$  arise only when the component fields  $F_{L,R}$  are developed the nonzero vacuum expectation values. Another fact which is more suggestive in our opinion is that the gauge freedom of the residual system remained in our disposal allows one to put the gauge in which  $\lambda^\pm = 0$ . This is well-known unitary gauge which always can be achieved in any theory with the spontaneously broken gauge symmetry.

Returning the SFs (29), (30) back into the system (24), (25) we obtain the ordinary Liouville equation *only*

$$\tilde{\partial}_{--} \tilde{\partial}_{++} \tilde{u} = e^{2\tilde{u}} \tilde{F}_L \tilde{F}_R. \quad (31)$$

Two remainder Eqs. (25) are satisfied identically due to the constraints (30).

## 4 General solution

Let us consider shortly the problem of construction of general solution of the Eqs. (2), (3). It is obvious that this solution can be obtained directly from the residual Eq. (31). Indeed, we know from [1] that the corresponding solution can be written in form

$$\begin{aligned} e^{-2\tilde{u}(\tilde{\xi}^{\pm\pm})} &= \frac{1}{2} \tilde{r}_m^{++}(\tilde{\xi}^{--}) \tilde{l}^{--m}(\tilde{\xi}^{++}), \\ \tilde{F}_L(\tilde{\xi}^{++}) &= \tilde{l}_m^{--}(\tilde{\xi}^{++}) \tilde{\partial}_{++} \tilde{l}^m(\tilde{\xi}^{++}), \\ \tilde{F}_R(\tilde{\xi}^{--}) &= \tilde{r}_m^{++}(\tilde{\xi}^{--}) \tilde{\partial}_{--} \tilde{r}^m(\tilde{\xi}^{--}), \end{aligned} \quad (32)$$

where the left(right)-moving Lorentz harmonics are normalized as follows

$$\tilde{l}_m^{++} \tilde{l}^{m++} = 0, \quad \tilde{l}_m^{--} \tilde{l}^{m--} = 0, \quad \tilde{l}_m \tilde{l}^{m\pm\pm} = 0, \quad (33)$$

$$\tilde{l}_m^{-} \tilde{l}^{m++} = 2, \quad \tilde{l}_m \tilde{l}^m = -1. \quad (34)$$

Performing here the change of the variables inverse relative to (18) one can always reach the general solution of the Eqs. (2) and (23) in terms of harmonic SFs restricted by the constraints

$$\begin{aligned} l_m^{\pm\pm,0}(\xi^{++}, \eta^+) &= \tilde{l}_m^{\pm\pm,0}(\tilde{\xi}^{++}), \\ r_m^{\pm\pm,0}(\xi^{--}, \eta^-) &= \tilde{l}_m^{\pm\pm,0}(\tilde{\xi}^{--}). \end{aligned} \quad (35)$$

## 5 Conclusion

Thus we have demonstrated that owing to the relaxation of the gauge fixing auxiliary equation of motion (3)  $\Rightarrow$  (23) we obtain the ordinary Liouville equation (31) instead of SF Eq. (2). This is very important result because it clarifies the general method of construction of the superstring-inspired nonlinear equations of motion in the case of arbitrary space-time dimension  $D$  starting directly from the corresponding equation of motion in the bosonic sector. Indeed, let us suppose that we have known the linear realization of the supergroup  $G$  in the worldsheet superspace  $n = (p, p)$ ,  $p = 1, 2, 4, 8$ , which describes the corresponding superconformal transformations. Decomposing of an arbitrary element of this group onto the product of two elements, i.e. proper chosen coset space and that of the stability subgroup  $G = KG_0$ , we can always obtain the suitable nonlinear realization of  $n = (p, p)$  superconformal symmetry following closely to the pattern of the Section 2. Then the sought for form of the SF equation of motion could be deduced with the help of the transformations which are inverse relative to  $\zeta = \tilde{K}(\tilde{\zeta})$ .

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## References

- [1] I. Bandos, E. Ivanov, A. Kapustnikov and S. Ulanov *J. Math. Phys.* **40** (1999) 5203–5223 (**hep-th/9810038**).



- [2] F. Lund and T. Regge *Phys. Rev.* **D 14** (1976) 1524;  
R. Omnes *Nucl. Phys.* **B 149** (1979) 269;  
B.M. Barbashov and V.V. Nesterenko *Commun. Math. Phys.* **78** (1981) 499;  
A. Zheltukhin *Sov. J. Nucl. Phys. (Yadern.Fiz.)* **33** (1981) 1723; *Theor. Mat. Phys.* **52** (1982) 73; *Phys. Lett.* **B 116** (1982) 147; *Theor. Mat. Phys.* **56** (1983) 230.
- [3] I. Bandos, P. Pasti, D. Sorokin, M. Tonin and D. Volkov *Nucl. Phys.* **B 446** (1995) 79-119 (**hep-th/9501113**).
- [4] I. Bandos *Phys. Lett.* **B 338** (1996) 35.
- [5] I. Bandos, D. Sorokin and D. Volkov *Phys. Lett.* **B 372** (1996) 77-82.
- [6] M. Chaichian and P. Kulish *Phys. Lett.* **B 78** (1978) 413.
- [7] E. Ivanov and S. Krivonos *J. Physics* **A 17** (1984) L671.
- [8] E. Ivanov and A. Kapustnikov *J. Physics* **A 11** (1978) 2375; *J. Physics* **G 8** (1982) 167; *Phys. Lett.* **B 252** (1990) 212-220; *Int. J. Mod. Phys.* **A 7** (1992) 2153.
- [9] E. Ivanov and A. Kapustnikov *Nucl. Phys.* **B 333** (1990) 439.
- [10] S. Coleman, J. Wess and B. Zumino *Phys. Rev.* **177** (1969) 2239;  
C. Callan, S. Coleman, J. Wess and B. Zumino *Phys. Rev.* **177** (1969) 2247;  
D. Volkov *J. Elem. Part. Atom. Nucl.* **4** (1973) 3;  
V. Ogievsky *in Proc. X Winter School of Theor. Physics* (Wroclaw, 1974), **vol. 1**, p. 117.